Let $X$ be our favorite Banach space of continuous functions on $\mathbb{R}^n$ (e.g. $C^m$, $C^{m,\alpha}$, $W^{m,p}$). Given a real-valued function $f$ defined on an (arbitrary) given set $E$ in $\mathbb{R}^n$, we ask: How can we decide whether $f$ extends to a function $F$ in $X$? If such an $F$ exists, then how small can we take its norm? What can we say about the derivatives of $F$? Can we take $F$ to depend linearly on $f$?

What if the set $E$ is finite? Can we compute an $F$ whose norm in $X$ has the smallest possible order of magnitude? How many computer operations does it take? What if we ask only that $F$ agree approximately with $f$ on $E$? What if we are allowed to discard a few points of $E$ as "outliers"; which points should we discard?

A fundamental starting point for the above is the classical Whitney extension theorem.

The results are joint work with Arie Israel, Bo’az Klartag, Garving (Kevin) Luli, and Pavel Shvartsman.